ISSN: 2449 - 0539
BAYERO JOURNAL OF ENGINEERING AND TECHNOLOGY VOL 19 NO. 3 AUGUST 2024 PP. 38- 46

MODELING AND PARAMETER IDENTIFICATION OF A TWO-WHEELED INVERTED PENDULUM ROBOT

Fatima Dalhatu Sani[1*], **Isyaku Abubakar**[1]

[1]Department of Electrical Engineering, Bayero University, Kano, PMB 3011, Nigeria
Email of Corresponding Author: fdsani.ele@buk.edu.ng

ABSTRACT

This paper presents a dynamic model and parameter identification for a two-wheeled inverted pendulum (TWIP) robot. The work is aimed at obtaining an accurate mathematical model of the TWIP and obtaining its parameters. The mathematical model is developed using the Euler-Lagrange method. Then, the parameters of the system are identified using measurements and computations. The nonlinearity and instability of the TWIP is evaluated through simulations of the model in SIMULINK. The simulation results show that the developed model represents the actual system in terms of nonlinearity and instability. Thus, the model can be used to develop a controller for the nonlinear plant.

Keywords: Two-wheeled Inverted Pendulum (TWIP) robot; Lagrange modeling technique; parameter identification; generalized coordinates, tilt angle

1. INTRODUCTION

A two-wheeled mobile robot, also called a two-wheeled inverted pendulum robot (TWIP) or self-balancing robot, is a control system which has its basis on the inverted pendulum. The robot has been used in various applications; for transportation of humans to short distances, like the Segway[1]; for medical purposes where it has helped people with disabilities to climb stairs, and in gardening[2] . Self-balancing robots are also excellent for teaching and testing advanced control systems. The TWIP is an eco-friendly system since it does not use fossil fuels.

The TWIP, being based on the inverted pendulum, is highly unstable, nonlinear and underactuated[3]. These features of its dynamics make it a very attractive plant to control engineering researchers [4], right from the modelling of its dynamics, to the design of controllers for its digital and physical realization. Researchers have been concerned with obtaining models that most closely describe the dynamics of the TWIP using different methods and approaches. They have also been occupied with designing different types of controllers for its operations. Being a nonlinear system, some control design approaches employing its linearized equivalent have been studied and applied. The use of a linearized model requires correct parameter identification. The parameters of the TWIP must be identified correctly in order to fit the developed model and enable the design of an adequate controller.

In the literature, TWIPs have been modeled using different techniques: In [5], the Euler-Lagrange modeling technique was used on a TWIP driven by DC motors. The model was then linearised and a state space model developed. In [2], a co-axes driven TWIP was developed, after which the mathematical model was obtained using the Lagrange method. Other researchers that used the Euler-Lagrange method include [6-17]. The Euler-Lagrange method appears to be the most popular method of modeling the TWIP as there is no need for interactive forces. Newtonian mechanics is another method used to model TWIPs. The authors in [18] developed a dynamic model of the Scorpio self-balancing robot using this method. In[19] , various control strategies were tested on a TWIP which was modelled using Newtonian mechanics. Another method of modeling the TWIP is Kane's method. Researchers in [20, 21] used Kane's method to derive a dynamic model of a TWIP.

In addition to an accurate model, the parameters of the TWIP must be identified correctly in order to fit the developed model and enable design of an adequate controller. The main contribution of this paper is to identify the parameters of the TWIP robot and to demonstrate the inherent instability and nonlinearity of the robot. The dynamical equations of the TWIP are obtained using Euler-Lagrange modeling. The parameters of the robot are then identified using measurements and computations. A SIMULINK model is then developed based on the equations. The model is then simulated and results are presented.

The rest of the paper is organized as follows: Section 2 presents the dynamic mathematical model. Section 3 presents the identification of the parameters. In section 4, the model is simulated and results discussed. In section 5, conclusions are presented.

2. DYNAMIC MODEL

The TWIP used in this work is a product of Arduino, shown in Figure 1, while Figure 2 shows its schematic diagram [22]. The TWIP is a highly nonlinear system which requires an accurate model for the development of an adequate controller. Thus, a mathematical model of the TWIP is developed using the Euler-Lagrange method.

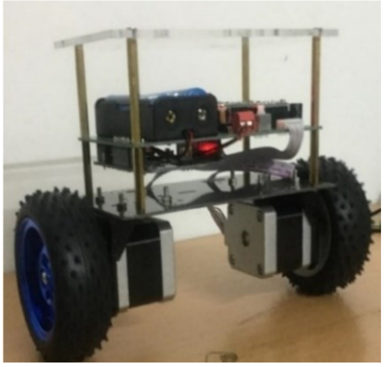

Figure 1 TWIP

The model is based on the following assumptions:

i) The robot is a rigid body and does not get distorted while moving
ii) The tilt angle is measured from the vertical
iii) Both wheels are exactly the same
iv) There is no sliding between the wheels and the ground
v) The dynamics of the motors are neglected (since the time constants of the electric motors is small compared to the time constant of the TWIP)
vi) Friction and cornering forces are neglected

Table 1 shows the parameters and variables used in this work.

The Lagrangian equation is given by

$$L = T - V \quad \ldots \tag{1}$$

Where

$L =$ The Lagrangian function,
T= The total kinetic energy (KE), and
V =The total potential energy

The equations for evaluating the total kinetic energy,T, the total potential energy V, and hence the Lagrangian function are derived in the following subsections.

2.1 Total Kinetic Energy

The total kinetic energy of a body is the sum of its translational kinetic energy T_t, and its rotational kinetic energy T_r. i.e.,

$$T = T_t + T_r \tag{2}$$

Translational Kinetic energy, T_t

The translational kinetic energy is a function of the mass of a body and its translational velocity.

For the main body m, its translational kinetic energy is given by:

$$T_{t1} = 0.5mv_1^2 \quad \ldots \quad (3)$$

Where v_1 is its translational velocity, defined as the square root of the sum of the squares of its horizontal velocity v_{1h} and its vertical velocity v_{1v}. i.e.,

$$v_1 = \sqrt{v_{1h}^2 + v_{1v}^2}$$

(4)

The horizontal and vertical velocities are obtained from the horizontal and vertical displacements of the main body as follows:

The horizontal displacement of the body is

$$x_c = R\varphi + Lsin\Theta \quad \ldots \quad (5)$$

Therefore, the horizontal velocity is

$$v_{1h} = \dot{x}_c = R\dot{\varphi} + \dot{\theta}Lcos\theta \ldots \quad (6)$$

The vertical position of the centre of gravity (CoG) of the main body is

$$Z_c = Lcos\theta + R \quad \ldots \quad (7)$$

So, the vertical velocity is

$$v_{1v} = \dot{z}_c = -\dot{\theta}Lsin\theta \quad \ldots \quad (8)$$

$$v_1 = \sqrt{(R\dot{\varphi} + \dot{\theta}Lcos\theta)^2 + \left(-L\dot{\theta}sin\theta\right)^2}$$

$$= \sqrt{R^2\dot{\varphi}^2 + 2RL\dot{\varphi}\dot{\theta}\cos\theta + L^2\dot{\theta}^2} \ \ldots \quad (9)$$

∴, the translational kinetic energy of the main body is

$$T_{t1} = 0.5mR^2\dot{\varphi}^2 + mR\dot{\varphi}L\dot{\theta}cos\theta + 0.5mL^2\dot{\theta}^2 \quad \ldots$$

(10)

The translational kinetic energy of the two wheels, T_{t2} is obtained as follows:

The horizontal displacement of one wheel is

$$x_m = R\varphi \quad (11)$$

Therefore, the horizontal velocity of one wheel is

$$v_2 = \dot{x}_m = R\dot{\varphi} \quad (12)$$

$$T_{t2} = 0.5m_w v_2^2 = m_w R^2\dot{\varphi}^2 \ \ldots \quad (13)$$

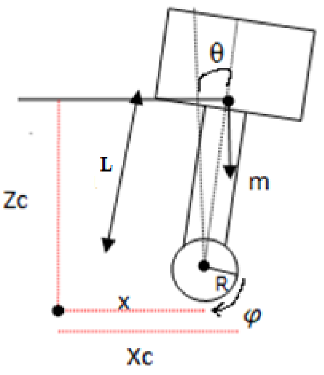

Figure 2 Schematic diagram of TWIP

Table 1: Parameters and Variables

Symbol	Quantity/Parameter
m	mass of main body
m_w	mass of one wheel
v_1	horizontal velocity of the main body of the TWIP
v_2	horizontal velocity of one wheel
x_c	horizontal position of the centre of gravity, CoG, of the body with respect to an initial position O
Z_c	The vertical position of the centre of gravity (CoG) of the TWIP body
x	horizontal position of the centre of one wheel with respect to an initial position
R	Radius of one wheel
L	Length between Centre of gravity and center of wheels
g	Acceleration due to gravity
□	Height from CoG to the ground
θ	Tilt angle
φ	Angular position of wheels
ω	angular velocity of main body
J	Moment of inertia of the main body
J_w	Moment of inertia of the wheels

Therefore, the total translational kinetic energy of the TWIP is

$$T_t = 0.5mR^2\dot{\varphi}^2 + mR\dot{\varphi}L\dot{\theta}cos\theta + 0.5mL^2\dot{\theta}^2 + m_w R^2\dot{\varphi}^2 \quad \ldots \quad (14)$$

Rotational Kinetic Energy, T_r

The rotational kinetic energy is the sum of the rotational kinetic energies of the main body and that of the two wheels. It is derived as follows:

The rotational kinetic energy of the main body is

$T_{r1} = 0.5J\omega^2$

The rotational KE of both wheels is

$T_{r2} = J_w\omega_w^2$

Therefore, the total rotational kinetic energy of the TWIP is

$$T_r = T_{r1} + T_{r2} = 0.5J\omega^2 + J_w\omega_w^2 \quad \ldots \quad (15)$$

Combining equations (14) and (15), the total kinetic energy of the TWIP is

$$T = T_t + T_r = 0.5mR^2\dot{\varphi}^2 + mR\dot{\varphi}L\dot{\theta}cos\theta + 0.5mL^2\dot{\theta}^2 + m_wR^2\dot{\varphi}^2 + 0.5J\omega^2 + J_w\omega_w^2 \quad \ldots (16)$$

2.2 Total Potential Energy

The total potential energy of the TWIP is given by

$$V = mgh \quad \ldots \quad (17)$$

From Figure 2, the height $h = Z_c = Lcos\theta + R$ . . . (18)

Therefore, $V = mg(Lcos\theta + R)$ . . . (19)

2.3 Lagrangian Function

Combining equations (16) and (19), the Lagrangian is obtained as

$$L = 0.5mR^2\dot{\varphi}^2 + mR\dot{\varphi}L\dot{\theta}cos\theta + 0.5mL^2\dot{\theta}^2 + m_wR^2\dot{\varphi}^2 + 0.5J\omega^2 + J_w\omega_w^2 - mg(Lcos\theta + R) \ldots (20)$$

The general form of Lagrange's equation is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \quad \ldots \quad (21)$$

where

$i = ith$ degree of freedom (DOF) of system

q_i is a generalised coordinate

Q_i is the generalized force associated with a generalized coordinate q_i

In this work, the generalized coordinates are:

q_1 representing the coordinate along the displacement angle of the (*wheel of the*) robot, φ

q_2 representing the coordinate along the tilt angle of the robot, θ

So the q vector is

$$q = [q_1 q_2] = [\,\varphi\ \theta] \quad \ldots \quad (22)$$

Let the generalized force associated with the φ coordinate be μ, and that of θ be β

So, the general form of Lagrange's equation for φ is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = \mu \quad \ldots \quad (23)$$

From equation (20)

$$\frac{\partial L}{\partial \varphi} = 0 \quad \ldots \quad (24)$$

Also,

$$\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = mR^2\dot{\varphi} + 2m_wR^2\dot{\varphi} + 2J_w\dot{\varphi} + mRL\dot{\theta}cos\theta \quad (25)$$

So,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = mR^2\ddot{\varphi} + 2m_wR^2\ddot{\varphi} + 2J_w\ddot{\varphi} - mRL\dot{\theta}^2sin\theta + mRL\ddot{\theta}cos\theta \quad \ldots \quad (26)$$

Therefore, the Lagrangian for φ =

$$\ddot{\varphi}(mR^2 + 2m_wR^2 + 2J_w) - mRL\dot{\theta}^2sin\theta + mRL\ddot{\theta}cos\theta = \mu \quad (27)$$

The Lagrangian for the second generalized coordinate θ is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \beta \quad \ldots \quad (28)$$

$$\frac{\partial L}{\partial \theta} = -mRL\dot{\varphi}\dot{\theta}sin\theta + mgLsin\theta \quad \ldots \quad (29)$$

$$\frac{\partial L}{\partial \dot{\theta}} = mRL\dot{\varphi}cos\theta + mL^2\dot{\theta} + J\dot{\theta} \quad \ldots \quad (30)$$

This means

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = -mRL\dot{\varphi}\dot{\theta}sin\theta + mRL\ddot{\varphi}cos\theta + mL^2\ddot{\theta} + J\ddot{\theta} \quad (31)$$

Therefore, the Lagrangian for the θ coordinate is

$$-mRL\dot{\varphi}\dot{\theta}sin\theta + mRL\ddot{\varphi}cos\theta + mL^2\ddot{\theta} + J\ddot{\theta} - \left(-mRL\dot{\varphi}\dot{\theta}sin\theta + mgLsin\theta\right) = \beta \quad \ldots \quad (32)$$

$$mRL\ddot{\varphi}cos\theta + \ddot{\theta}(mL^2 + J) - mgLsin\theta = \beta \quad \ldots \quad (33)$$

In matrix form, the Lagrangians are

$$\begin{bmatrix} mR^2 + 2m_wR^2 + 2J_w & mRLcos\theta \\ mRLcos\theta & mL^2 + J \end{bmatrix}\begin{bmatrix} \ddot{\varphi} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & -mRL\dot{\theta}sin\theta \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ -mgLsin\theta \end{bmatrix} = \begin{bmatrix} \mu \\ \beta \end{bmatrix} \quad (34)$$

3. PARAMETER IDENTIFICATION

The parameters identified are the mass of the body m, mass of one wheel m_w, the radius of the wheels R, the length between the centre of gravity and the centre of the wheels R, and the moment of inertia of the main body, J.

Mass of the body m

This was measured using a weighing balance in the lab. It was found to be 1.034Kg.

Mass of one wheel m_w

This was measured using a weighing balance in the lab and was found to be 0.0496Kg

Radius of the wheels R

The radius was measured using a meter rule and was found to be 0.0425m.

Length between centre of gravity and the center of wheels B

This was measured using a meter rule and found to be 0.022m.

Moment of inertia of the main body, J:

The main body of the TWIP is rectangular, with breadth a and height b.

For a rectangular body, the moment of inertia about the x-axis is given as

$$J = \frac{1}{12}m(b^2 + a^2)$$

Where b is the height and a the breadth

b was measured using a metre rule and found to be 0.13m, while a was found to be 0.082m.

$$J = \frac{1}{12}(1.034)(0.13^2 + 0.082^2) \quad ... \quad (35)$$

$$J = 0.001Kgm^2 \; . . .$$

Moment of inertia of the wheels, J_w:

$$J_w = \frac{1}{2}m_w R^2 \quad . . . \quad (36)$$

$$J_w = \frac{1}{2}(0.0496)(0.0425)^2 = 4.48e - 5 \quad . . . \quad (37)$$

Next, the generalized forces μ and β are substituted by known parameters.

The parameters identified are summarized in Table 2

Table 2: Parameters of the TWIP

Parameter	Symbol	Value	Unit
Mass of the TWIP body	m	1.034	kg
Mass of one wheel	m_w	0.0496	kg
Radius of one wheel	R	0.0425	m
Length between Centre of gravity and center of wheels	B	0.022	m
Acceleration due to gravity	g	10	ms^{-2}
Tilt angle	θ		rad
Angular position of wheels	φ		
Moment of inertia of the main body	J	0.001	kgm^2
Moment of inertia of the wheels	J_w	4.48e-5	kgm^2

The torques on the wheels are: right wheel torque τ_1 and left wheel torque τ_2. When the robot carries out translational motion, the wheels move in opposite directions-one wheel clockwise, the other anticlockwise

So, generalized force μ due to q_1, ie φ, is

$$\mu = \tau_1 + \tau_2 \quad \dots \quad (39)$$

For q_2, ie θ, when the robot tilts, the wheels have to move in a direction opposite that of the fall of the TWIP to balance it. So,

$$\beta = -(\tau_1 + \tau_2) \quad \dots \quad (40)$$

Substituting for μ and β in equations (26) and (32) gives

$$\ddot{\varphi}(mR^2 + 2m_wR^2 + 2J_w) - mRL\dot{\theta}^2 sin\theta + mRL\ddot{\theta}cos\theta = \tau_1 + \tau_2 \quad . . . \quad (41)$$

$$mRL\ddot{\varphi}cos\theta + \ddot{\theta}(mL^2 + J) - mgLsin\theta = -(\tau_1 + \tau_2) \quad (42)$$

From equations 41 and 42, we obtain equations (43) and (44)

$$\ddot{\theta} = \frac{[-(\tau_1 + \tau_2) - mRL\ddot{\varphi}cos\theta + mgLsin\theta]}{[mL^2 + J]} \quad ... (43)$$

$$\ddot{\varphi} = \frac{\tau_1 + \tau_2 + mRL\dot{\theta}^2 sin\theta - mRL\ddot{\theta}cos\theta}{mR^2 + 2m_wR^2 + 2J_w} \quad ... \quad 44)$$

4. SIMULATION RESULTS

This section presents the results of the simulations. The nonlinear dynamics and instability of the TWIP were simulated using MATLAB/SIMULINK. The parameters of the robot as given in Table 1 are used in the simulations. The dynamic equations are as obtained in equations 43 and 44.

The simulations were carried out under different conditions as shown below

1) **Without external input:** This is when the torque is set to zero

a) With a Positive initial tilt angle, set to 0.001 radians.

Figure 4a shows the response of the TWIP. With the tilt angle set to positive 0.001 rad, it can be seen that the TWIP falls in about 0.9s in the positive direction. The robot falls in the positive direction because the tilt angle is initialized to a positive value. Thus, the highly unstable nature of the TWIP is observed. Figure 4b shows the position of the TWIP. It moves back and forth before falling.

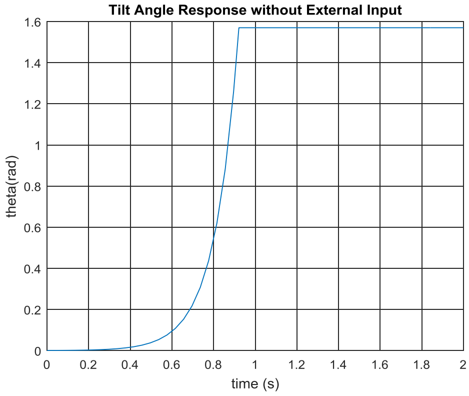

Figure 4a Tilt angle response when tilt angle is set to 0.001 rad

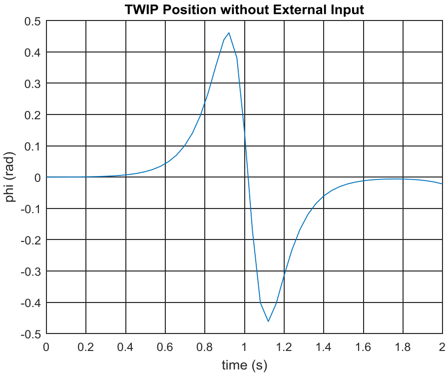

Figure 4b TWIP Position when tilt angle is 0.001 rad

b) With a negative initial tilt angle, set to -0.001radians.

Figure 5a shows the response of the TWIP while Figure 5b shows the position of the TWIP. It is seen that the TWIP now falls in the opposite direction

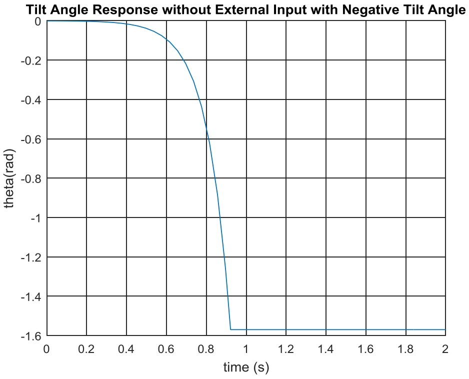

Figure 5a Tilt angle response when tilt angle is set to -0.001 rad

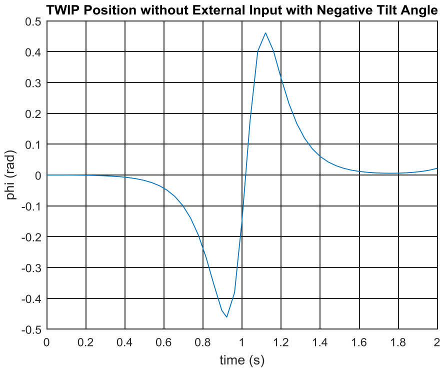

Figure 5b TWIP Position vs Time when the tilt angle is -0.001 rad

2) With a step input of 0.02N/m^2 Figure 6a shows the response of the tilt angle with a step input of 0.02N/m^2. It can be observed that the TWIP falls in about 0.12s in the positive direction. Figure 6b shows the position of the TWIP. With the torque applied, the robot falls and the tires continue to rotate.

These results show the inherent nonlinearity and instability of the TWIP.

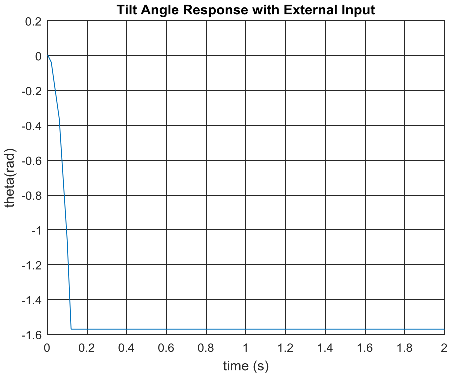

Figure 6a Tilt angle response when tilt angle is set to 0.001 rad

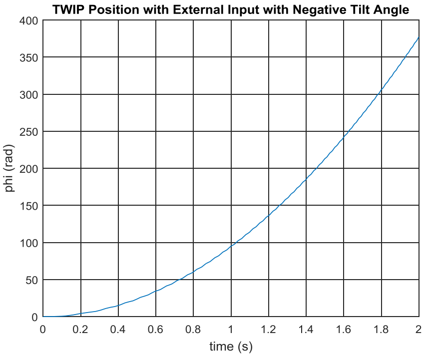

Figure 6b TWIP Position vs Time when the tilt angle is 0.001 radians

5. CONCLUSION

A mathematical model of the self-balancing robot has been developed and simulated using SIMULINK in MATLAB. The parameters of the robot were identified and measured. Using the model and parameters obtained, an accurate controller can be developed. From the results obtained in the simulations, it has been shown that the TWIP is an inherently unstable system which falls in the direction of tilt as soon as it is released. From the simulation results, it is seen that the highly nonlinear and unstable nature of the TWIP has been demonstrated.

6. ACKNOWLEDGEMENTS

The contributions of the following people are highly appreciated: Prof. Mustapha Muhammad, Assoc. Prof. Amir Bature, Dr. Binta Usman and Dr. Hassan Bashir.

REFERENCES

[1] N. M. Mohd Noor, S. A. A. M. Halid and R. M. Wahab "Study The Performance Of Two-Wheeled Balancing Mobile Robot Using Fuzzy Pd Controller," *Jurnal Mekanikal,* 2023.

[2] H. R. Memarbashi, "Design and Parametric Control of Co-Axes Driven Two-Wheeled Balancing Robot," Master's, Mechatronics, Massey University, New Zealand, 2010.

[3] Predoi, "Stabilization Of A Two-Wheeled Inverted Pendulum Robot," *UPB,* 2024.

[4] D. M. S. Ravindra K. Munje , Deepak P. Kadam, "Control of a Two-Wheeled Inverted Pendulum using Integral Sliding Mode," *International Journal of Mechanical Engineering,* 2022.

[5] P. Frankovský, L. Dominik, A. Gmiterko, I. Virgala, P. Kurylo, and O. Perminova, "Modeling of Two-Wheeled Self-Balancing Robot Driven by DC Gearmotors," *International Journal of Applied Mechanics and Engineering,* vol. 22, pp. 739-747, 2017.

[6] X. G. Fuquan Dai, Shigong Jiang, Wenzeng Guob, Yubai Liu, "A two-wheeled inverted pendulum robot with friction compensation," *Elsevier,* August 2015.

[7] J. W. Qingwen Qian, Zhe Wang, "A novel configuration of two-wheeled self-balancing robot," in *Lecture Notes in Electrical Engineering. 327, (2015)*, ed, 2017, pp. 207-219.

[8] P. T. B. a. M. S. J., "Balancing of Two Wheeled Inverted Pendulum using SOSMC and Validation on Lego EV3" presented at the 1st IEEE International Conference on Power Electronics, Intelligent Control and Energy Systems (ICPEICES-2016) India, 2016.

[9] O. K. Sayidmarie, "<Design and real-time control of a new structure two-wheeled robot.pdf>," PhD, The University of Sheffield, 2016.

[10] N. Esmaeili, A. Alfi, and H. Khosravi, "Balancing and Trajectory Tracking of Two-Wheeled Mobile Robot Using Backstepping Sliding Mode Control: Design and Experiments," *Journal of Intelligent & Robotic Systems,* vol. 87, pp. 601-613, 2017.

[11] E. Philip, Golluri, S., "Implementation of an Autonomous Self-Balancing Robot Using Cascaded PID Strategy," presented at the 6th International Conference on Control, Automation and Robotics, 2020.

[12] A. A. Bature, " Control Of Two Wheeled Inverted Pendulum Mobile Robot Using Composite Nonlinear Feedback Control Approach", Universiti Teknologi Malaysia, 2016.

[13] A. U. a. O. Aydogdu, "Adaptive control of two-wheeled mobile balance robot capable to adapt different surfaces using a novel artificial neural network–based real-time switching dynamic controller," *International Journal of Advanced Robotic Systems* 2017.

[14] N. S. Roger van Rensburg, Lambert Trénoras, Yskandar Hamam & Eric Monacelli, Eric, "Stability and enhancement analysis of a modelled self-balancing verticalized mobility aid using optimal control techniques," *African Journal of Science, Technology, Innovation and Development,* 2017.

[15] S. B. Amir A. Bature, Mohamed. N. Ahmad, Mustapha Muhammad, " A Comparison of Controllers for Balancing Two Wheeled Inverted Pendulum Robot" *International Journal of Mechanical & Mechatronics Engineering IJMME-IJENS,* vol. IJMME-IJENS Vol:14 No:03, June 2014 2014.

[16] S. K. a. S. Kwon, "Dynamic Modeling of a Two-wheeled Inverted Pendulum Balancing Mobile Robot," *International Journal of Control, Automation, and Systems,* 2015.

[17] K. Nader, D. Sarsri, and T. Zhang, "Modelling and Control of a Two-Wheel Inverted Pendulum Using Fuzzy-PID-Modified State Feedback," *Journal of Robotics,* vol. 2023, pp. 1-13, 2023.

[18] S. Kim and S. Kwon, "Dynamic modeling of a two-wheeled inverted pendulum balancing mobile robot," *International Journal of Control, Automation and Systems,* vol. 13, pp. 926-933, 2015.

[19] "Control strategies for inverted pendulum: A comparative analysis of linear, nonlinear, and artificial intelligence approaches," 2024.

[20] S. B. Mustapha Muhammad, M. N. Ahmad, S. W. Nawawi and A. A. Bature, "Multiple Operating Points Model-Based Control of a Two-Wheeled Inverted Pendulum Mobile Robot," *International Journal of Mechanical & Mechatronics Engineering IJMME-IJENS Vol:13 No:05,* 2013.

[21] M. N. A. a. J. H. S. O. S. W. Nawawi, "Development of a Two-Wheeled Inverted Pendulum Mobile Robot" presented at the The 5th Student Conference on Research and Development -SCOReD, Malaysia, 2007.

[22] J. G. Ye Ding, Mie Kunio, "Modeling, Simulation and Fabrication of a Balancing Robot," *Advanced System Dynamics and Control,* 2012.