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OPTIMIZATION OF DISTRIBUTION SYSTEMS INCORPORATING DISTRIBUTED GENERATION AND BATTERY ENERGY STORAGE ASSETS

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ABSTRACT

The introduction of new technologies like Distributed Generation (DG), Battery Energy Storage Systems (BESS) and Smart Meters, called collectively Distributed Energy Resources (DER), introduce new challenges and opportunities for the Distribution Network Operator (DNO). Thus, proper planning is required to ensure that a cost-effective solution is obtained that can improve on the system affordability. This paper deploys a multi-stage stochastic algorithm to minimize the cost of the Distribution System (DS) which consists of the Distribution Network (DN) and Customer costs. This paper proposes a central planning approach where the decision variables are the sizing, siting, and operation of BESS, along with the sizing and siting of DG and Capacity of the distribution lines. The algorithm estimates the optimal BESS's energy storage capacity and the maximum hourly delivery capacity. The customers' objective function is the capacity and energy payments minus the savings and revenue from the DG. The DNO's goal is to minimize the cost of the distribution network and BESS's investments and operational costs. Since the model has a long-term approach to planning, the paper presents the implementation of Expected Value as decision criterion to handle the uncertainty and the impact of the remuneration rules for DG (i.e., Solar PV) on customers' and DN's cost are calculated. The results show that the incorporation of Solar PV brings savings to the customers while the BESS in addition can provide modest improvements in network performance.

Index Terms-- Battery Energy Storage Systems (BESS), Distributed Generation (DG), Mixed-Integer Linear Programming, Network Planning, Solar PV, Stochastic Programming.

1. NOMENCLATURE

Parameters

CAP_{price}^{DG}	Investment price associated with DG (£/kW).
CAP_{ene}^{BESS} CAP_{cap}^{BESS}	Investment price in energy storage and inverter capacity of BESS (£/kW or £/kWh).
$k_{i,j}$	Relation between apparent and active power.
$LMP_{ene,h}^{market}$	Local marginal price at hour *h* (£/kWh).
$O\&M_{price}^{DG}$ $O\&M_{price,ene}^{BESS}$ $O\&M_{price,cap}^{BESS}$	Operation and maintenance (O&M) cost of DG and BESS. The sub-index "*ene*" and "*cap"*in BESS parameters means capacity and energy respectively (£/kWh or £/kW).
$P_i^{DG,lim}$	It is a regulatory limit of power delivered to the network by DG (kWh).
$P_{i,h}^d$	Active power demand at bus *i*, at hour *h* (kW).
pf_i^{DG}	Operational limits of the power factor of DG.
$Q_{i,h}^d$	Reactive power demand at bus *i*, at hour *h* (kVAr).
$r_{i,j}$; $x_{i,j}$	Resistant and reactance of the branch *i-j* (p.u).
$VA_i^{max/min}$	Max/Min voltage angle at bus *i* at hour *h* (radian).
$VM_i^{max/min}$	Max/Min voltage magnitude at bus *i* at hour *h* (p.u).
$\alpha_{i,j}^{fix/var}$	Fix and variable cost of the branch *i-j* (£ or £/kVA).
$\eta_{ro-trip+}$	BESS's efficiency of the charging process.
$\eta_{ro-trip-}$	BESS's efficiency of the discharging process.
$\phi_{ene/cap,h}^{tar}$	Energy/capacity tariff associated with the consumption/demand (£/kWh or £/kW).
$\phi_{exe/low,h}^{tar,DG}$	Price for energy delivered by DG when production is higher (*exe*) or lower (*low*) than the demand (£/kWh).
π_h^{DG}	Maximum hourly production from DG per kW installed (kWh/kW).

Variables

$CAPEX_i^{DG}$	Investment cost of DG at node i (£).
$CAPEX_i^{BESS}$	Investment cost of BESS at node i (£).
$C_{i,tb}^d$	Customer's capacity demand at time block *tb* (kW).
CI_i^{DG}	DG's capacity installed at bus *i* (kW).
$CI_{sto,i}^{BESS}$	BESS's storage capacity installed at bus *i* (kWh).
$CI_{del,i}^{BESS}$	BEE's inverter size at bus *i* (kW).
DS	Distribution System cost including DN and customer (£).
IE_i^{Cus}	Revenue from DG's production delivered to the network (£).
TC_i^{Cus}	Capacity customer payment (£).
TE_i^{Cus}	Energy customer payment (£).
$NC_{i,j}$	Cost of the branch *i-j* (£).
$O\&M_i^{BESS}$ $O\&M_i^{DG}$	Operation and maintenance (O&M) cost of BESS and DG (£) at bus *i*.
P_i^{DG}	Capacity installed of the DG at bus *i* (kW).
$P_{i,h}^{DG}$	Active power produced by the DG at bus *i*, at hour *h* (kWh).
$P_{i,h}^{DG,low/exe}$	Active power produced by the DG which is lower/higher than the demand at bus *i*, at hour *h* (kWh).
$P_{i,h}^d$	Customer's active power demand at bus *i*, at hour *h* (kWh).
$P_{i,h}^{BESS+}$	Active power taken from the network (kWh).

$P_{i,h}^{BESS-}$	Active power delivered to the network (kWh).
$P_{i,j,h}^{line}$, $Q_{i,j,h}^{line}$	Active and reactive power flow in the branch i-j (kWh/kVArh).
$P_{i,j,h}^{losses}$, $Q_{i,j,h}^{losses}$	Active and reactive power losses in the line i-j (kWh).
$SoC_{i,h}^{BESS}$	State of Charge SoC at bus i, at hour h (kWh).
$S_{max,i,j}^{line}$	Maximum capacity of the branch i-j (kWh).
$P_{i,j,h}^{+}$;$P_{i,j,h}^{-}$	Auxiliar variables to handle the $\lvert S_{max,i,j}^{line} \rvert$(kWh).
SE_{i}^{Cus}	Customer savings because of consumption from the integration of DG (£).
$VM_{i,h}$	Voltage magnitude at bus i at hour h (p.u.).
$VA_{i,h}$	Voltage angle at bus i at hour h (radians).
$\mu_{i,j,h}$	Binary variable to manage the absolute value of the flow.
$\sigma_{i,h}$	Binary variable which is equal to 1 if the power output of the DG at hour t has exceeded the demand.

2. INTRODUCTION

Distribution Systems (DS) composed of both the customer and Distribution Network (DN), are experiencing significant changes in their roles. The Distribution Network Operator DNO's aim is to minimize the DS cost. The introduction of new technologies like Distributed Generation (DG), Battery Energy Storage Systems (BESS) and Smart Meters, called collectively Distributed Energy Resources (DER), introduce new challenges and opportunities for the DNO.

There are several references about the optimization of the DS to reduce cost, considering the above-mentioned technologies. (Asensio, Meneses de Quevedo, Muñoz-Delgado, & Contreras, 2018) present a holistic model including the joint optimal network expansion and the location of DG. The model is applied to an isolated system in Canary Island, Spain. In (Pal, Jabr, & Agalgaonkar, 2014), a model to control the voltage in the DN with high penetration of photovoltaic systems is presented. The goal is to optimize the voltage profile while reducing the number of operations of voltage control devices. The work reported by (Zuboa, Mokryani, & Abd-Alhameed, 2018) propose the optimization of the social welfare of the distribution system under high DG penetration and the model is tested using 16-buses distribution case system. (Singh, Bishnoi, & Meena, 2020) presents a heuristic technique to minimize losses in distribution system using DG and Transformers' on-load tap changer - OLTC; (Hong, Zhao, Zhang, Cui, & Tian, 2020) has a similar approach. A market for reactive power in distribution systems is presented in (Homaee, Zakariazadeh, & Jadid, 2019) and (Bai, Wang, Wang, Chen, & Li, 2018) showing the results of eventual clearing process. The work by (Ma, Dasenbrock, Töbermann, & Braun, 2019) emphasised the relevance of energy losses reduction due to DG penetrations. A novel indicator is proposed to avoid exhaustive power flow calculations. The average error tends to be low but in some specific cases can be up to 25%. A stochastic algorithm to optimize generation size an allocation is proposed in (Yin, Li, & Yu, 2022) considering the intermittency in the Solar PV production and the load variations. In (Stennikov, Barakhtenko, Mayorov, Sokolov, & Zhou , 2022), an intelligent integrated energy system with active customers operating with renewable energy sources is proposed. The most interesting contribution of this paper is the multi-agent approach where the interest of every participant is properly modelled. The paper considers both heat and electricity needs. The optimal allocation of distributed generation and capacitor banks is presented in (Pereira, y otros, 2022). The stochastic dependence between the generation sources is considered. Researchers in (Shang, y otros, 2020) proposes a dynamic dispatch of BESS using a multi-period stochastic optimization problem. A combination of several technologies (PV, wind, diesel) is tested.

This paper appreciates contributions made by the literature on BESS modelling. However, it is necessary to improve the model of the BESS, to consider the uncertainties of the long-term and to use a more realistic tariff model to represent the customer cost. A comparison of the most relevant literature available and the paper's proposal is presented in Table 1, which shows that key improvements compared with previous studies are the stochastic considerations and the BESS model for calculation of energy and capacity.

Table 1. Comparison of Existing Literature with the Proposed Model

Reference	**Customer tariff model**		**Network Type**	**Network cost**	**DG Model**		**BESS**		**Deterministic or stochastic**
	Capacity or energy	**DG Consumer profile**			**Saving and revenues**	**Reactive power**	**Capacity**	**Energy**	
(Asensio, Meneses de Quevedo, Muñoz-Delgado, & Contreras, 2018)	Energy	No	Radial	Yes	No	No	No	Yes	Deterministic
(Pal, Jabr, & Agalgaonkar, 2014)	Energy	No	Radial and mesh	No	Yes	Yes	No	No	Deterministic
(Zuboa, Mokryani, & Abd-Alhameed, 2018)	Energy	No	Radial	No	No	Yes	No	No	Deterministic
(Singh, Bishnoi, & Meena, 2020), (Hong, Zhao, Zhang, Cui, & Tian, 2020)	No	No	Radial	No	No	Yes	No	No	Deterministic
(Homaee, Zakariazadeh, & Jadid, 2019), (Bai, Wang, Wang, Chen, &	No	No	Radial	No	No	Yes	No	No	Deterministic

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Reference	Customer tariff model		Network Type	Network cost	DG Model		BESS		Deterministic or stochastic
	Capacity or energy	DG Consumer profile			Saving and revenues	Reactive power	Capacity	Energy	
Li, 2018)									
(Ma, Dasenbrock, Többermann, & Braun, 2019)	No	No	Radial	No	No	Yes	No	No	Deterministic
(Yin, Li, & Yu, 2022)	No	No	Radial	Yes	Yes	Yes	No	No	Stochastic
(Stennikov, Barakhtenko, Mayorov, Sokolov, & Zhou , 2022)	Yes	No	No	No	Yes	No	No	No	Deterministic
(Pereira, y otros, 2022)	No	No	No	No	Yes	Yes	No	No	Stochastic
(Shang, y otros, 2020)	Yes	No	No	No	No	No	Yes	Yes	Stochastic
Proposed	Capacity and Energy	Yes	Radial and mesh	Yes	Yes	Yes	Yes	Yes	Stochastic

The main contributions of this paper are summarized as follows:

- The algorithm proposed handles the uncertainty in the long-term parameters using a two-stage stochastic model minimizing the expected value of the objective function.
- The BESS's model has a relevant flexibility where the size of the inverter (capacity) and the size of the batteries (energy) are computed as separate variables, reducing the total cost.
- The model proposed utilizes the actual Time-of-Use (ToU) tariff regime faced by the customer considering capacity and energy charges.
- The model computes the benefits of DG considering the actual regulation, where the customers' savings and the revenue from DG depends on the match between the production- and demand-profile. This relationship is made for both energy and demand charges.

This paper presents a centralized planning approach to minimize the cost of the Distribution System using BESS and DG. The centralized planning approach enables one single entity to take decision about the operation and investment in all the above-mentioned technologies. The optimization model decides the best size and location of DG and BESS as well as their hourly optimal operation. The network is represented by a linearization of the well-known AC power equations.

This paper is organized as follows: Problem formulation is presented in section III, describing the objective function, along with the Techno-economical constraints of the BESS, DG, and the power flow equations. Section IV describes the case used and presents the main considerations of the technique. Section V presents and discusses important results and outcomes. Finally, the conclusions of the work are presented in section VI.

3. PROBLEM FORMULATION

This paper presents a centralized planning approach to minimize the Distribution System Cost (customer and Distribution Network) when deploying new technologies, specifically BESS and DG. The optimization problem considers the operations and maintenance cost (O&M) and the investment cost (CAPEX). The model minimizes annual operation cost, where the CAPEX is annualized considering 15 years lifetime, typical of BESS and DGs. The customer demand is modelled using hourly-resolution daily load profile. There are two generation sources: energy from the wholesale market, delivered at the HV/MV substation, and the energy provided by DG. The DG technology considered here is PV, since the cost is very affordable even at small scale being very suitable as distributed generation (Agency, 2020). DG size and location is determined by the algorithm and the production is limited by the installed capacity (kW) and the traditional Solar PV production profile. In this section, the mathematical formulation of the problem along with the recast of the equations, to sort-out non-linearities is presented.

a) Objective fuction

The objective function aims to minimize the cost of the Distribution System (DS) including the customer's energy cost and the distribution network long-term cost (DN).The customer cost includes the energy bill (energy and capacity) costs, savings, and revenues from DG. The DN's costs encompass the network and BESS. The total cost of the DS is depicted in (1).

$$Min \sum_i \{TE_i^{Cus} + TC_i^{Cus} - SE_i^{Cus} - IE_i^{Cus}\} - \sum_i \{O\&M_i^{DG} + CAPEX_i^{DG}\} + \sum_{i,j} NC_{i,j} + \sum_i \{O\&M_i^{BESS} + CAPEX_i^{BESS}\} \quad (1)$$

The first term includes cost, savings and revenue faced by the customer. The second one is O&M and investment cost of DG. The last two terms are the DN costs, the network cost, and O&M and CAPEX of the BESS.

b) The customer cost function

The customers are required to pay according to the energy and capacity-demands. The unit price of these services is defined by the regulatory authority; therefore, those values are parameters in the formulation. Equations (2) and (3) show the energy and capacity payments respectively. In (2) the tariff electricity price $\phi_{ene,h}^{tar}$ can change hourly, this is usually referred to as time-of-use-tariff (ToU) model. Traditionally, the ToU considers three-time blocks prices for the energy charge (peak, base and valley). In (3), one capacity payment is calculated, using the maximum demand by each time block (*tb*).

$$TE_i^{Cus} = \sum_h P_{i,h}^d \times \phi_{ene,h}^{tar} \;\; h \in T \tag{2}$$

$$TC_i^{Cus} = \sum_{tb} C_{i,tb}^d \times \phi_{cap,tb}^{tar} \;\; tb \in T_{block} \tag{3}$$

The customers can decide to invest in a DG, to achieve savings in their energy payment. Depending on the regulation framework, the energy generated can be discounted by the energy consumption (net-metering) or paid depending on whether the production exceed the consumption. Equations (4) and (5) computes the revenue for the energy production which is lower than the consumption, since the energy used in this calculation $P_{i,h}^{DG,low}$ is always lower than the energy consumption. Equation (6) computes the revenue when the DG's production is higher than the demand.

$$SE_i^{Cus} = \sum_h P_{i,h}^{DG,low} \times \phi_{low,h}^{tar,DG} \quad h \in T \tag{4}$$

$$P_{i,h}^{DG,low} \leq P_{i,h}^d \;\; h \in T \tag{5}$$

$$IE_i^{Cus} = \sum_h P_{i,h}^{DG,exe} \times \phi_{exe,h}^{tar,DG} \;\; h \in T \tag{6}$$

Therefore, the energy production from the DG is paid depending on whether the DG's production is lower or higher than the demand. In Equations (7) and (8) an auxiliary binary variable $\sigma_{i,t}$ is introduced to ensure the energy production from DG is first assigned to $P_{i,h}^{DG,low}$ and only when the production is higher than the consumption this is allocated to $P_{i,h}^{DG,exe}$. Equation (9) limits the energy delivered to the network to any regulatory limit $P_i^{DG,lim}$. The DG's investment and operation costs are represented in (10) and (11).

$$P_{i,h}^{DG,low} + P_{i,h}^{DG,exe} = P_{i,h}^{DG} \forall h \in T \tag{7}$$

$$P_{i,h}^d \times \sigma_{i,t} \leq P_{i,h}^{DG,low} \forall h \in T \tag{8}$$

$$P_{i,h}^{DG,exe} \leq \sigma_{i,h} \times P_i^{DG,lim} \forall h \in T \tag{9}$$

$$CAPEX_i^{DG} = CI_i^{DG} \times CAP_{price}^{DG} \; \forall i \in N \tag{10}$$

$$O\&M_i^{DG} = CI_i^{DG} \times O\&M_{price}^{DG} \; \forall i \in N \tag{11}$$

c) The distribution costs

The DNO decides when and where the network requires expansion or reinforcements to attend the demand or new assets to improve the performance. Hence, to invest in BESS and distribution lines assets are DNO's decisions.

The investment cost in BESS (12), depends on both the store capacity i.e., the size of the battery $CI_{sto,i}^{BESS}$ and the hourly delivery capacity i.e. the size of the inverter $CI_{del,i}^{BESS}$.

$$CAPEX_i^{BESS} = CI_{sto,i}^{BESS} + CAP_{ene}^{BESS} + CI_{del,i}^{BESS} \times CAP_{cap}^{BESS} \forall i \in N \tag{12}$$

The operation and maintenance cost of the BESS, calculated in (13) depends on the size of the inverter $CI_{del,i}^{BESS}$ and the energy taken and delivered from the system $P_{i,h}^{BESS}$.

$$O\&M_i^{BESS} = \sum_t \{P_{i,h}^{BESS} \times O\&M_{price,ene}^{BESS}\} + CI_{del,i}^{BESS} \times O\&M_{price,cap}^{BESS} \forall i \in N \tag{13}$$

d) Power flow equations

In this section, the constraints associated with the power balance and technical limits are presented in (14)-(15). Equations (14) and (15) set the hourly bus balance of active and reactive power respectively, for all busbars.

$$P_{i,h}^{DG} - P_{i,h}^{BESS+} + P_{i,h}^{BESS-} - P_{i,h}^d - \sum_j \frac{P_{i,j,h}^{losses}}{2} = \sum_j P_{i,j,h} \;\; \forall j \in N_i \tag{14}$$

$$Q_{i,h}^{DG} - Q_{i,h}^{BESS+} + Q_{i,h}^{BESS-} - Q_{i,h}^d - \sum_j \frac{Q_{i,j,h}^{losses}}{2} = \sum_j Q_{i,j,h} \;\; \forall j \in N_i \tag{15}$$

Eq. (16) and (17) calculate the power flow through the branches$P_{i,j,h}$,$Q_{i,j,h}$and the voltage magnitude $VM_{i,h}$and voltage angle $VA_{i,h}$. Note the linearization of the AC power flow equations for ease of computation.

$$P_{i,j,h} = g_{i,j}(VM_{i,h} - VM_{j,h}) - b_{i,j}(VA_{i,h} - VA_{j,h}) \ \forall h \in T;\ \forall j \in N_i \tag{16}$$

$$Q_{i,j,h} = -b_{i,j}(VM_{i,h} - VM_{j,h}) - g_{i,j}(VA_{i,h} - VA_{j,h}) \ \forall h \in T;\ \forall j \in N_i \tag{17}$$

The steps taken in linearization of power losses is shown in Appendix, making a piece-wise linearization, and using integer variables. Further information about the linearization strategies in power systems can be found in (Yang, y otros, 2019), (Zhang, Vittal, Quintero, & Heydt, 2013). The limits in the power flow by branch and voltage magnitude and angle are shown in equation (18) through (20).

$$0 \le |P_{i,j,h} \times k_{i,j}| \le S_{i,j}^{max} \ \forall h \in T;\ \forall i,j \in N \tag{18}$$

$$VM_j^{min} \le VM_{j,h} \le VM_j^{max} \quad \forall h \in T;\ \forall j \in N \tag{19}$$

$$VA_j^{min} \le VA_{j,h} \le VA_j^{max} \quad \forall h \in T;\ \forall j \in N \tag{20}$$

e) Distributed Generation(DG) model

The energy generated from the DG is limited by the capacity installed and the availability of the natural resources (the Sun). The DG reactive power is constrained by the inverter specifications. Here, the DG is considered to operate at the nominal 0.8 lagging or leading power factor.

$$0 \le P_{i,h}^{DG} \le CI_i^{DG} \times \pi_h^{DG} \ \ \forall h \in T \tag{21}$$

$$|Q_{i,h}^{DG}| \le \tan\{\cos^{-1}(pf_i^{DG})\} \times P_i^{DG} \ \ \forall h \in T \tag{22}$$

f) Battery Energy Storage System (BESS) model

The technical constraints associated with the operation of the BESS are shown in (23) to (25). The energy balance in the State-of-Charge is represented by (23). Equations (24) and (25) set limits in the maximum energy storage and the hourly energy delivered.

$$SoC_{i,h}^{BESS} = SoC_{i,h-1}^{BESS} + \frac{P_{i,h-1}^{BESS+}}{(1+\eta_{ro-trip+})} - P_{i,h-1}^{BESS-} \times (1+\eta_{ro-trip-}) \forall h \in T; i \in N \tag{23}$$

$$0 \le SoC_{i,h}^{BESS} \le CI_{sto,i}^{BESS} \forall h \in T;\ i \in N \tag{24}$$

$$P_{i,h}^{BESS+} \le CI_{del,h}^{BESS} \ \ \forall h \in T;\ \forall i \in N \tag{25}$$

$$P_{i,h}^{BESS-} \le CI_{del,h}^{BESS} \ \ \forall h \in T;\ \forall i \in N \tag{26}$$

g) Network Cost

The model proposed here allows the DNO to regulate on the capacity of the branches when it is required and profitable to do so. Equation (27) computes the branch cost using a linear function of the maximum flow. The offset cost $\alpha_{i,j}^{fix}$ of the Equation (27) represents the supporting structure (trench/pole and isolators) and the variable cost $\alpha_{i,j}^{var}$ is associated with the cable size.Since the flow can be positive of negative and the cost is always positive, it is necessary to calculates the absolute value of flow $|P_{i,j,h}|$, this is done using (28), (29) and (30).

$$NC_{i,j} = \alpha_{i,j}^{fix} + (P_{i,j,h}^{+} + P_{i,j,h}^{-}) \times k_{i,j} \times \alpha_{i,j}^{var} \forall\ i,j \in N \tag{27}$$

$$P_{i,j,h}^{+} - P_{i,j,h}^{-} = P_{i,j,h} \ \ \forall\ i,j \in N; h \in T \tag{28}$$

$$P_{i,j,h}^{+} \le \mu_{i,j,h} \times P^{max} \ \ \forall\ i,j \in N;\ \forall\ h \in T \tag{29}$$

$$P_{i,j,h}^{-} \le (1-\mu_{i,j,h}) \times P^{max} \ \ \forall\ i,j \in N;\ \ \forall\ h \in T \tag{30}$$

It is noteworthy, that Equation (21) considers that the DG is supplying at its maximum level, which is a not always the case because of the intermittence in the irradiance. Thus, to consider the unavailability of the DG, a complete set of equations neglecting the contribution of DG can be included in the model, however, this is not presented for reasons of simplicity. The flow for every branch is calculated considering the worst condition, with and without DG (31).

$$max\{P_{i,j,h}^{With\ DG}, P_{i,j,h}^{Without\ DG}\} = P_{i,j,h} \ \ \forall\ i,j \in N; h \in T \tag{31}$$

4. CASE STUDY

A case study of a 16-bus UK generic distribution system (UKGDS) is selected to apply the proposed algorithm. The UKGDS is a 33kV radial MV distribution network having 16 buses and 18 lines, as shown in Figure 1. The DG and BESS size and location is computed using the proposed method.

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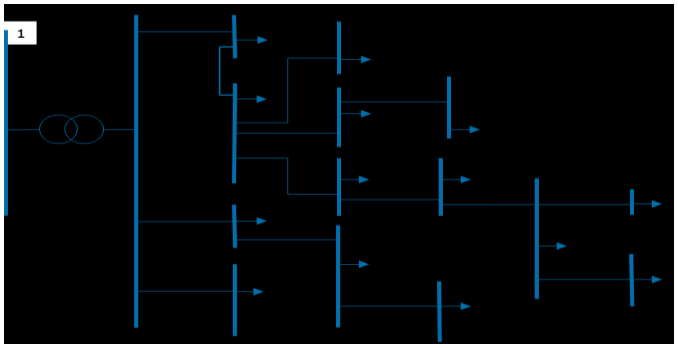

Figure 1. 16-bus UKGDS single-line diagram

Network parameters and geographical location are available in (Imperial College, s.f.). The tariff used in the simulations, energy (£/MWh) and capacity (£/MW-month) prices, are shown in Table 2. These prices show the behaviour of the Mexican energy market of Valle de Mexico in the daily market (Centro Nacional de Control de Energia CENACE, 2021) and the capacity market (Energia-CENACE, s.f.).

Table 2. Customer Tariff-Energy and Capacity

ToU	Peak	Base	Valley
$\boldsymbol{\phi^{tar}_{ene,h}}$	55.00	50.00	45.00
$\boldsymbol{\phi^{tar}_{cap,tb}}$	4,000.00	0.00	0.00

5. RESULTS AND DISCUSSION

As shown in the problem formulation, the power flow equations are a linearization of the actual power flow equations. It is important to assess the accuracy of the model after the linearization. Table 3 shows the average, minimum and maximum error in the main variables of the network comparing the linearization vs AC power flows.

TABLE 3. AVERAGE/MIN/MAX ERROR BY VARIABLE

Error (%)	Voltage magnitude	Voltage angle	Losses	Power flow by branch
Average	0.229%	8.575%	0.123%	-0.57%
Max	0.452%	16.010%	14.47%	-34.56%
Min	0.000%	0.000%	0.06%	0.05%

The results show that the errors are acceptable in planning tools. A detailed analysis of error and how to improve it is provided in Annex. To understand the impact of each technology in the DS cost, the results are presented sequentially to identify the synergies.

a) *Impact of DG*

The impact of the DG's deployment on the distribution system depends on the size and location of the assets and demand profile, among other factors. Not all the buses can deploy DG because either the customer is not interested or is not suitable. In this case study, DG will not be installed at buses 4, 6, 8, 10, 12 and 14. In the remaining nodes, the algorithm decides the capacity to be installed.

The CAPEX of the DG is set to 1,000 £/kW, considering a lifetime of 15 years. The O&M cost of both the energy and the capacity are 0.0012£/kWh and 10£/kW-year respectively. Four scenarios were defined for the payment of the net energy delivered to the network $\phi^{tar,DG}_{exe,h} = \{0,10,20,30\}$£/MWh.

Table 4 show the customers' energy cost, Total Customer Cost (TCC), Distribution Network's cost, Network Cost (NC) and the total Distribution System's cost (Total Cost) of the four scenarios described before, quantities in millions £/year. TCC goes from 4.58 when there is not DG until 4.10 in scenario 4, i.e., 11%cost reduction. The revenue from selling energy surplus to the network IE^{Cus}clearly increases with the price $\phi^{tar,DG}_{exe,h}$. The TC^{Cus}presents little changes because the energy production from DG do not reduce the capacity demand during the peak time.

The objective function (DS cost) varies from 5.17 to 4.57 which represents 11.61 % cost reduction.

Table 4. Results Minimizing Customer's Cost

Cost Variables	Before DG	After DG deployment			
		E1	**E2**	**E3**	**E4**
$\boldsymbol{TE^{Cus}}$	4.01	4.01	4.01	4.01	4.01
$\boldsymbol{TC_i^{Cus}}$	0.58	0.57	0.57	0.57	0.57
$\boldsymbol{IE^{Cus}}$	0.00	0.00	0.01	0.15	0.23
$\boldsymbol{SE^{Cus}}$	0.00	0.89	0.89	1.07	1.07
$\boldsymbol{CAPEX^{DG}}$	0.00	0.51	0.51	0.78	0.78
$\boldsymbol{O\&M^{DG}}$	0.00	0.02	0.02	0.03	0.03
Total Customer Cost (TCC)	**4.58**	**4.22**	**4.21**	**4.18**	**4.10**
Network Cost (NC)	0.52	0.48	0.48	0.48	0.48
Total Cost	**5.11**	**4.70**	**4.69**	**4.65**	**4.57**

Two points here requires additional thoughts. NC cost reduction eventually will be passed to customer, via a lower distribution charge, bringing additional savings to customer. The energy sold by DG to the network is borne by the local retailer at $\phi^{tar,DG}_{exe,h}$. If the retailer energy price is calculated as a pass-through, and the buying price increases, customer eventually will get a higher energy tariff. Therefore, a more compressive model is required to consider how the cost of the DN, and the retailer finally impacts the customer costs.

b) *Joint impact of BESS and DG in the operational cost*

The BESS's CAPEX prices and O&M parameters used in these simulations can be found in (Jannesara, Sedighia, Savaghebib, & Guerrero, 2018). In this work for reasons of simplification it is assumed that the benefit of the suitable

BESS far outweighs its cost and thus considered negligible. The total annual network cost reduction is 90,763.25£/year, i.e., 1.94%. The reduction is particularly small because of the network cost used here: the fixed cost is about 70% of the line cost. Despite in some cases the maximum flow reduction is relevant, the total line cost changes so little because the variable cost stands at 30%.

The network cost used here were provided by local utilities, unfortunately those are confidential. (Arriaga, 2013) provides useful information about distribution network cost. However, the cost of the BESS required to achieve such improvement is 3.16 million £/year, which makes the investment not profitable.

To make rewarding the integration of the BESS in the DS, besides reducing the network cost, BESS could find other revenue sources such as selling ancillary services to the wholesale market or flexibility services to DNO. It is worth noticing that from the optimization process the ratio of $\sum_i CI_{sto,i}^{BESS} / \sum_i CI_{del,i}^{BESS}$ is 2.84, which suggests that the inverter size must be considerably lower than the total storage capacity of the batteries.

c) *Taking decisions under uncertainty*

In a long-term model there are always parameters with uncertainty, in this paper the future DG remuneration's $\phi_{exe/low,h}^{tar,DG}$ is considered a regulatory uncertainty. To minimize the DS cost in such case, it is necessary to manage a multi-stage stochastic problem. In the first stage model it is necessary to calculate the variables that do not change with the realization of the uncertain parameters, for instance the investments, and in the second stage the variables that change in every realization, for instance the revenue from selling energy. In this paper by simplicity only one single parameter is considered stochastic $\phi_{low,h}^{tar,DG}$ which is modelled using a discrete probability density function (pdf) with twenty realizations $\Omega = \{w_1, \dots, w_{20}\}$. Therefore, the savings from the DG production $SE_{i,w}^{Cus}$ changes with every realization.

The price scenarios for the energy production $\phi_{low,h}^{tar,DG}$, when it is lower than the consumption, will be proportional to the consumption tariff $\phi_{ene,h}^{tar}$. With this consideration, the price scenarios are $\{w_1 = 1.38 \times \phi_{ene,h}^{tar}, \dots, w_{20} = 0.75 \times \phi_{ene,h}^{tar}\}$. Besides modelling the uncertainty, it is necessary to define risk criteria, where the must traditional are Expected Value (E$\{x\}$), Value-at-risk (VaR) or Conditional Value-at-risk CVaR. Expected Value is the objective function weighted by probability of every realization. This paper uses Expected Value and the objective function (1) becomes (32). E$\{SE_i^{Cus}\}$ and E$\{TC\}$represent the Expected Value of the savings and the total cost. The simulation results are shown in Figure 2. The Expected Value of the savings is 77 k£/year, the DG capacity installed is 632 kW and the customer's savings for every realization are depicted in Figure 2. Every variable that depends on the stochastic parameter becomes probabilistic variable and the algorithm computes its probability density function (pdf).

$$Min\ \mathrm{E}\{TC\} = \sum_i \{TE_i^{Cus} + TC_i^{Cus} - \mathrm{E}\{SE_i^{Cus}\} - IE_i^{Cus}\} + \cdots \quad (32)$$

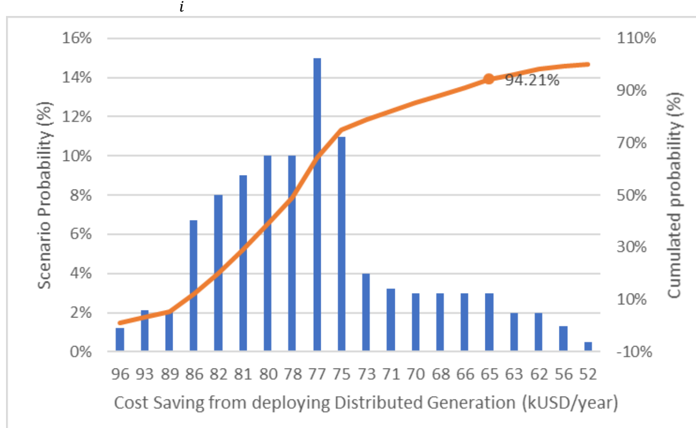

Figure 2. Probability Density Function of the Objective function

The PDF provides useful information about the distribution of the risk, for instance, there is a probability of 94.21% of getting savings lower than 64.86k£/year or conversely a 5.79% of having higher savings.

6. CONCLUSION

This paper presented a model to integrate BESS and DG assets in Distribution Systems using a central planning approach, where one single entity decides how to operate the network and the investments in DG and BESS. The model assesses and quantifies the benefit for both customer and the DN. The simulations suggest DG and BESS bring important benefits to DS agents (consumers and network). DG is by far the technology that brings more benefits to the DS.

However, it is important to note that DG facilities are decided by the customers neglecting any DN benefit (Stennikov, Barakhtenko, Mayorov, Sokolov, & Zhou , 2022). In this sense, the Policy Maker must define the right signal to encourage the customer, in their DG deployment, to improve the global benefit while looking for their own interest.

The use of BESS can improve the performance of the system up to 1.94% but its cost is larger than the savings. Therefore, the BESS, providing only energy shifting service in the DN, are not profitable, hence additional service like regulations must be considered.

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APPENDIX

The linearization of the power losses calculation is described here. The losses in the actual power flow model are calculated by the square of the flow. Here, to avoid such non-linear representation, the square of the flow is calculated using a piece-wise linearization. Equation (33) calculates the value of small blocks $\Delta S_{i,j,h,b}$ in such way that sum of all blocks$b$ is equal to flow. The size of every block is limited to keep small resolution. Equations (34) and (35) calculates the active and reactive power losses respectively. ρ_b is the slope of the block b of function $\left|S_{i,j}\right|^2$. The number of blocks, and the slope for such blocks, should be enough to represent the highest flow in the model. Once defined the maximum expected flow in the system, the size of every block is defined, to keep a reasonable number of blocks.

$$\sum_b \Delta S_{i,j,h,b} = \{P^+_{i,j,h} - P^-_{i,j,h}\} \times k_{i,j} = |S_{i,j,h}| \tag{33}$$

$$\sum_b \{\Delta S_{i,j,h,b} \times \rho_b\} \times r_{i,j} = |S_{i,j,h}|^2 \times r_{i,j} = P^{losses}_{i,j,h} \tag{34}$$

$$\sum_b \{\Delta S_{i,j,h,b} \times \rho_b\} \times x_{i,j} = |S_{i,j,h}|^2 \times x_{i,j} = Q^{losses}_{i,j,h} \tag{35}$$

The results of the simulations in Table 5, show that losses error calculation is higher when the flow is small. The enumber of blocks b used in the linearization is the same for all lines, for instance 40 blocks. If the maximum flow is 12 MW, the size of every block is 12MW/40=300 kW. There are some blocks where the fow is significantly lower than this value.

Table 5. Error in Losses Calculation

Time	From bus	To bus	Active power flow		Error (%)
			GAMS	MATPOWER	
T1	1	2	5.22	5.24	0.48%
T1	3	4	2.58	2.59	0.61%
T1	2	3	3.78	3.80	0.58%
T1	4	6	0.19	0.20	3.22%
T1	4	5	0.43	0.43	0.73%
T1	6	7	0.08	0.08	5.01%
T1	4	8	0.56	0.56	0.56%
T1	8	9	0.26	0.26	1.69%
T1	9	10	0.22	0.23	3.20%
T1	10	11	0.07	0.07	5.56%
T1	10	12	0.05	0.05	2.82%
T1	2	13	1.30	1.30	0.13%
T1	2	14	0.01	0.01	-34.56%
T1	13	15	1.26	1.26	0.10%
T1	15	16	0.60	0.60	0.05%
System's total			**16.59**	**16.69**	**0.5778%**