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EFFECT OF SEED POINTS ON THE GEOMETRIC AND MECHANICAL PROPERTIES OF VORONOI FOAMS

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ABSTRACT

Foams are cellular materials that have unique mechanical properties. The geometric properties of these structures have been proven to be responsible for their unique mechanical properties. Accurately reproducing these geometric properties in modeling and simulation remains a challenge for researchers.This paper provides details on the geometric modeling techniques used to model foams and investigates the effects of the initial seed points on the geometric and mechanical properties. Unit cell repetition methods such as the Kelvin cell,and Gibson-Ashby cell as well as Voronoi-based models such as Poisson Voronoi, Hardcore Voronoi, and Laguerre Voronoi werepresented. The Voronoi models were further investigated, and their effectiveness in reproducing the geometric properties of foams and predicting their elastic properties was evaluated.Both the Poisson Voronoi and Hardcore Voronoi models overestimated the elastic properties by 169% and 25%, respectively.The Laguerre Voronoi model was the most accurate in predicting the elastic properties, with an absolute error of 6.5%.

Keywords: Geometric modeling; Foams; Elastic properties, Cell wall thickness; Voronoi tessellations; Voronoi foams

1. INTRODUCTION

Originally developed in the aerospace industry as a core for structural sandwich panels, metallic foams can be used in almost every lightweight and impact absorption engineering application, owing to their high strength-to-weight ratio.Recently, they have become salient materials in various engineering applications. This popularity of foams is attributed to their microstructure, which has been categorized as stochastic yet with some trends that can be captured using statistical distributions. A typical foam is characterized based on its cell types being open (open-cell foam) or closed (closed-cell foam), cell morphology, void density, and properties of the base material used during the foaming process.Cell morphology and void density are the leading parameters that affect the mechanical response of foams (Gibson & Ashby, 1982a, 1997a); hence, there is a need for an effective geometric model that captures these parameters.

Geometric modeling of metallic foams is not limited to computer-tomography (CT) reconstruction, although CT models produce good results, they are characterized by challenges in data acquisition and image analysis post-processing (Bici et al., 2017; Jeon et al., 2010; Youssef et al., 2005). The modeling strategies employed by researchers can be categorized into three types: unit cell replication, Voronoi-based tessellations, and unit cell subtraction.Unit cell replication involves the use of simple geometries arranged in a specific pattern to form an array.These unit cells include the cubic cell (Gibson & Ashby, 1997b), Kelvin cell (Gong et al., 2005), Gibson-Ashby cell(Gibson & Ashby, 1982b, 1997b), and

Weaire-Phelan cell(Weaire& Phelan, 1994), which were analyzed using classic beam and shell theory.Voronoi-based tessellations such as Poisson Voronoi (Zhang et al., 2016a, 2017), Hardcore Voronoi (Li et al., 2014; Song et al., 2010a; Tang et al., 2014), and Laguerre Voronoi tessellations are of interest to researchers because of the degree of randomness and because they simulate the manufacturing process of the foam, which is responsible for the microstructure and mechanical properties of the foam.

This study aims to compare the geometrical properties and mechanical responses of foam models produced by each modeling technique. First, the modeling strategies are described. Second, models were generated, and their properties were compared.Finally, a method for improving the accuracy of the model in predicting the mechanical response was proposed.

2. LITERATURE REVIEW

2.1. Overview of Foam Models

In the geometric modeling of metallic foams, unit cell replication is the most widely used method, owing to its simplicity and ease of numerical analysis. The process involves the arrangement of a simple cell unit in a particular

pattern to approximate the geometrical properties of the foam. On the other hand, Voronoi foam models are based on a space division technique that creates regions based on the position of the initial seed points.These models are discussed in the following subsections.

2.2. Kelvin Cell Model

Kelvin cells are of interest to researchers in many fields because of their minimal surface area per unit volume. It was discovered by Lord Kelvin in 1887 in the studies of bubble packing patterns. For more than a century, Kelvin cell has been believed to be a space-filling cell that minimizes the surface area per unit volume (Gibson & Ashby, 1997b). The unit cell (Fig. 1) consists of twenty-four vertices, eight hexagonal, and six quadrilateral faces. It has been used in studies of the mechanical properties of foams.

Song et al. (2010b) studied the dynamic crushing behavior of foams using the Kelvin model.The results indicated that increasing the cell irregularity improved both the densification strain energy and plateau stress; at a low impact velocity, the deformation bands first appeared in the middle of the foam.Weaire Phelan's cell model is a modified version of the Kelvin cell with two 12-sided polyhedra and six 14-sided polyhedra that has a lower surface area per unit volume than the Kelvin cell.

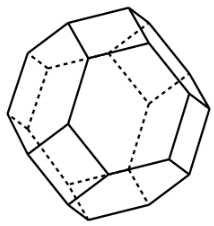

Fig. 1 Kelvin cell (Abdullahi et al., 2019)

2.3. Gibson-Ashby Model

Gibson and Ashby (1997b) created a new model that can be applied to both closed-cell and open-cell foams by choosing to close or open the faces of the unit cell.The model was developed from classic cube structures with 12 edges perpendicular to the adjacent pairs.The model (Fig. 2) is isotropic in all directions and has a uniform cross-section, making it useful for determining the linear elastic behavior of foams. The response for open-cell foams suggested by Gibson and Ashby (1997b) is indicated in equation (1)

$$\frac{E^*}{E_s} = C\left(\frac{\rho^*}{\rho_s}\right)^2 \qquad (1)$$

Where C is approximately equal to 1, E^* is Young's modulus of bulk foam, E_s is Young's modulus of the foam bulk material, ρ^* is the foam density, and $\rho_s$is the density of the bulk material.Using this model, the authors found that the relative density and bubble pore structure are factors that affect the mechanical properties of foams.

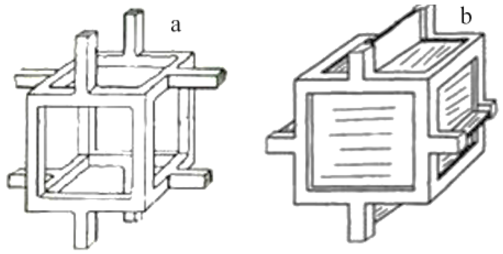

Fig. 2 Gibson-Ashby model(Gibson & Ashby, 1997b): (a) open-cell and (b) closed-cell

2.4. Weaire-Phelan Model

The Weaire-Phelan model is a widely used method for modeling foams. It is based on the concept of dividing space into cells with equal volumes, where the cells are irregular polyhedra that share faces and edges with neighboring cells.This model is particularly useful for simulating foams with large cells because it allows for an efficient and accurate representation of the foam structure. The Weaire-Phelan model has been used in numerous applications, including materials science, engineering, and architecture, and has provided valuable insights into the behavior of foams under various mechanical and environmental conditions(Weaire& Phelan, 1994).

2.4. Voronoi Models

Voronoi models are generated using a space division technique that creates regions from a group of points.It has the feature that any point placed within a region will be closer to the region's point than any other point in the space.The cells shorten the distance between themselves and the seeds. G. F. Voronoi proposed this method in 1905.

Let $P = \{p_1, p_2, p_3, \ldots..\}$ be a set of points in $\mathbb{R}^3$, a Voronoi cell (v-cell) corresponding to any point p_i in the set is defined as:

$$V_c(p_i) = \{p \in \mathbb{R}^3 : \|p - p_i\| \leq \|p - p_j\| \; \forall \, j \neq i\} \qquad (2)$$

Where V_c is the Voronoi cell corresponding to a point p_i, and p_j is any point outside the Voronoi cell. The Voronoi model was created in the field of computational geometry and is mostly utilized in the study of 2D foams (Andrews & Gibson, 2001; Torquato et al., 1998). The 3D Voronoi model simulates the growth process of voids in the manufacturing process and is geometrically and topologically identical to foams resulting from the growth process, making it a perfect model for studying the mechanical properties of real foams (Huang & Gibson, 2003).

3. MATERIALS AND METHODS

The material used was closed-cell ALPORAS foam with a 4.27 mm average cell diameter and a standard deviation of 0.99 mm.The dimensions of thespecimen were 35 mm × 35 mm × 35 mm.The cell wall thickness was distributed with a mean of 0.18 and a standard deviation of 0.92 mm. The relative density of the foam is 12.33%. All data for the foam specimens were extracted from the experimental work of Jang et al. (2015).Numerical models were then generated based on the extracted information using different Voronoi tessellation strategies to determine the effect of the seed generation method on geometric parameters and mechanical response.

Because we were interested in the effect of seed points in Voronoi foams, we considered three (3) types of Voronoi tessellations to generate and analyze the foam models.The Voronoi tessellations were Poisson, Hardcore, and Laguerre.The following subsections describe the tessellations and how they are used to generate the foam models in this study.

3.1. Poisson Voronoi Tessellations

Poisson Voronoi (PV) tessellation is a type of Voronoi in which the seed position is generated through a homogeneous Poisson point process, and the seeds and their respective Voronoi cells possess spatial randomness with no trend and share no interaction between seeds.Cell growth started from the seed position and stopped when the cell boundary came into contact with another cell boundary.Each point in P creates a Voronoi cell (v-cell), and all the v-cells combine to form a Voronoi diagram (VD), which divides the three-dimensional space into an array of convex, space-filling, nonoverlapping polyhedrons with planar faces.A Poisson Voronoi Diagram (PVD) generated with 100 seeds in a plane (2D) of 30 × 30 mm is shown in Fig. 3.

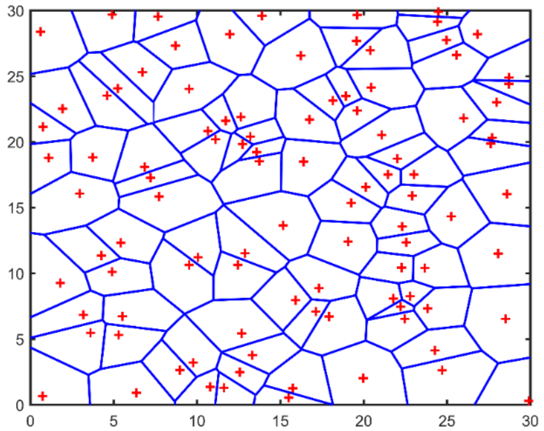

Fig. 3. Poisson Voronoi Diagram with 100 seeds.

The Poisson Voronoi Diagram (PVD) is characterized by a random cell size distribution with wide margins and a higher number of struts per vertex.In 3D, the PVD has an average number of faces per cell of 15.5 and an average number of edges per face of 5.2 (Wejrzanowski et al., 2013a).

In this work, Poisson Voronoi models are generated as follows: First, the number of seed (points) N is calculated as

$$N = \frac{V}{{}^{4}\!/_{3}\pi r^{3}} \qquad (3)$$

where V is the volume of the domain and r is the average radius of the sphere.N random points are then generated within the domain and fed to the Neper software, a polycrystal generation and meshing library (Quey et al., 2011), for the construction of the Voronoi tessellation.

3.2. Hardcore Voronoi Tessellations

Hardcore Voronoi tessellation is a special type of Voronoi tessellation in which the seed points are controlled to have a distance between each other, that is, each seed should be at a distance of at least a particular value known as the restricted distance.The algorithm generates random points sequentially, while testing the minimum restricted distance constraint.A point is accepted as a seed point for constructing a Voronoi diagram only if its distance from the previous points is greater than the defined minimum restricted distance (Falco et al., 2017; Falco et al., 2014; Falco, Jiang, et al., 2017a; Fritzen et al., 2009).It has been used in the geometric modeling of ceramic and metallic microstructures. The restricted distance (h) sometimes called *hardcore radius* is given by the expression

$$\|p_i - p_j\| = \sqrt{\sum_{k=1}^{3}\left(p_{i_k} - p_{j_k}\right)^2} > h$$

$$\forall(i,j) = 1,2,\ldots.NN : j \neq i \qquad (4)$$

Where p_i and p_j are points in the space, and $k$designates the x, y, and z componentsof the coordinates of the points. In 2D,k is limited to 2 (i.e., x and y). The Hardcore Voronoi approach provides control over the sphericity of cells.Sphericity refers to the ratio of the surface area of a sphere with the same volume to the cell's surface area. A Hardcore Voronoi model in a 2D plane is shown in Fig. 4.

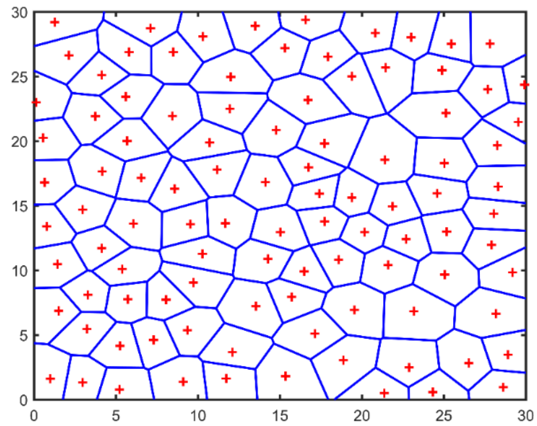

Fig. 4. A Hardcore Voronoi Diagram with 100 seeds and h=2mm

In this work, Hardcore Voronoi was constructed in a similar way to the Poisson Voronoi model, except that the random points were generated with a constraint on the *hardcore distance and* then fed to the Neper library.

3.3. Laguerre Voronoi Tessellations

Laguerre Voronoi tessellation (LVT) utilizes the introduction of weights to seed points.Researchers have used the packing of spheres with a predetermined volume distribution (Chen et al., 2015; Falco, Jiang et al., 2017b; Nie et al., 2017a, 2017b; Redenbach et al., 2012; Su et al., 2018; Wejrzanowski et al., 2013b)), wherethe center and radius of the spheres serve as the center and weight of the seed points, respectively.The distance between two neighboring cells, called *the power distance*, is perpendicular to the cell boundary plane.The power distance replaces the Euclidean distance in Poisson Voronoi. LVT can be described for any point p_i in the set $P = \{p_1, p_2, p_3, .., p_n\}$ a weighted r_i is provided to get a weight set $r = \{r_1, r_2, ..., r_n\}$, and the *power distance* between p_i and any point q is given by $d_L(p_i, q) = \{[d_v(p_i, q)]^2 - r_i^2\}^{\frac{1}{2}}$. A v-cell corresponding to a point p_i is defined as $v_L(p_i) = \{p | p \in R^3, d_L(p_i, q) < d_L(p, p_j), i \neq j\}$ (5)

Where v_l is the Laguerre Voronoi cell corresponding to a point p_i, and q is any point outside the Voronoi cell.The cells inherit the volume distributions of the spheres, which allows tuning of the geometric properties of the model by calibrating the input parameters, such as the volume distribution.A Laguerre Voronoi model in a two-dimensional (2D) plane is shown in Fig. 5.

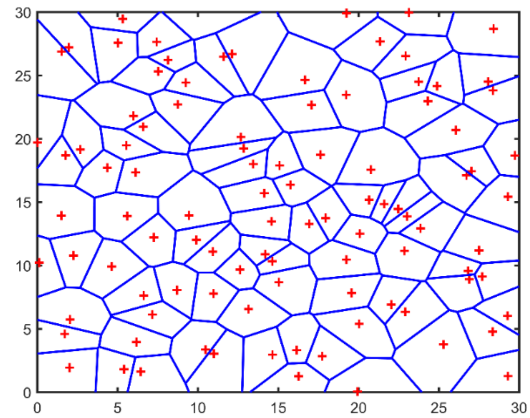

Fig. 5. Laguerre Voronoi diagram of 100 seeds with a logarithmic normal distribution (μ=4.38, σ=0.75)

In this work, the Laguerre Voronoi model is generated by first packing spheres with predetermined cell diameters into the domain.The drop and roll technique was used to generate spheres using the Molecular Dynamic software LIGGHTS (Kloss et al., 2012).The procedure is summarized as follows:

i. Generate spheres with predetermined cell diameter distribution
ii. Drop spheres into the domain using gravitational force.
iii. Consider collision forces with other spheres.

The centers and radii were fed to the NEPER library to construct the Laguerre Voronoi model.The entire process is illustrated inFig. 6

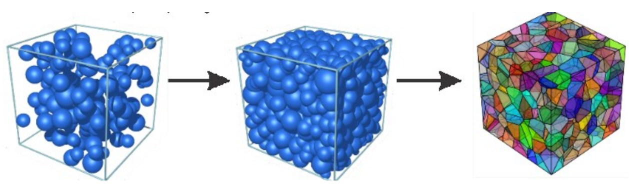

Fig. 6. Generation of Laguerre Voronoi

3.4. Numerical Simulation

The models were meshed with 4-node and 3-node shell elements using HyperMesh software (Altair, USA), and further subjected to a finite element compressive test to estimate the elastic properties in ABAQUS (Dassault Systems, USA).Each model was placed between two rigid plates; the bottom plate (red) was fixed, whereas the top plate (blue) was moved to a distance under quasi-static conditions, as shown in Fig. 7.

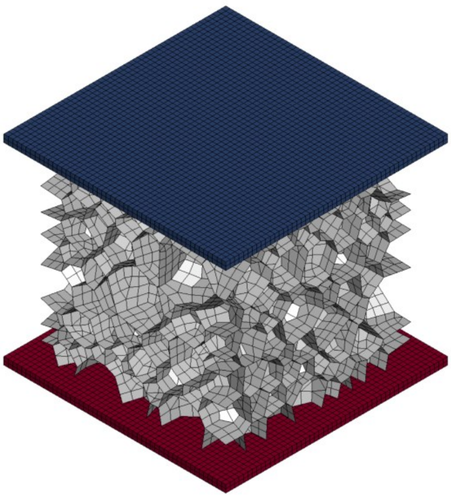

Fig. 7 Loading and boundary conditions

The relative density is controlled by assigning a uniform thickness to the shell elements, as shown in Equation 6.

$$t = \frac{V_s}{\sum S_i} \cdot \left(\frac{\rho^*}{\rho_s}\right) \quad (6)$$

Where V_s is the volume of the domain, Si is the surface area of the ith element, ρ^* is the foam density, and $\rho_s$is the bulk material density.The reaction forces and displacements of the top plate were recorded.The Young's modulus of the model was calculated as follows:

$$E^* = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\delta}{L}\right)} \quad (7)$$

Where F is the reaction force on the reference node of the rigid plate, A is the area of the top surface of the model, δ is the distance moved by the rigid plate, and L is the original length of the model.

4. RESULT AND DISCUSSION

4.1. Geometric Properties

The main parameters responsible for the mechanical behavior of foams after the relative density are the cell size (d_c), number of faces per cell (N_f), number of edges per face (N_e), and cell wall thickness (t) distribution. To identify these parameters, the foam models were constructed based on the three (3) types of Voronoi tessellations as shown in Fig. 8.The parameters extracted from the models are listed in Table 1.

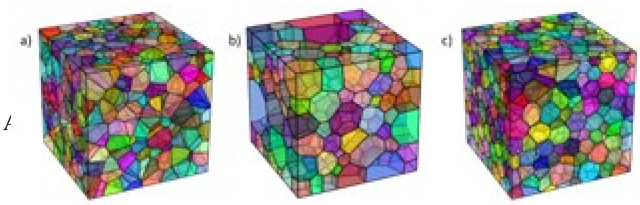

Fig. 8. The constructed (a) Poisson Voronoi (b) Hardcore Voronoi (c) Laguerre Voronoi models

Table 1 Geometric parameters of Voronoi-based models

Model	N_f	N_e	Surface Area (mm^2)	t (mm)	d_c (mm)
Foam specimen	14.00	5.00	-	0.1830	4.27
Poisson Voronoi	13.80	5.15	33612.80	0.1573	4.17
Hardcore Voronoi	13.90	5.15	31713.98	0.1667	4.04
Laguerre Voronoi	13.02	5.07	28593.76	0.1849	4.26

One interesting aspect that emerged from the analysis is that the Laguerre Voronoi models have the lowest absolute error compared to the experimental results except in N_f,which is due to the pruning of smallfaces employed by Neper software.This is attributed to the use of a pre-determined seed distribution and sphere packing that was introduced in the Laguerre Voronoi models to mimic the foam manufacturing process.The most striking observation that emerged from the analysis was the overestimation of the parameters using the Poisson Voronoi model.This is because seed points are generated randomly; therefore, there is no control over where the seed points are inserted, which might result in cell faces that are too small, thereby affecting the overall mechanical properties. Hardcore Voronoi models are somewhat in between the other two models. This is because they have control over the seed points' position to some degree, that is, they are not completely random (Poisson) or completely controlled (Laguerre).The results confirm that the seed generation method and positioning affect the geometric properties of the Voronoi foams.

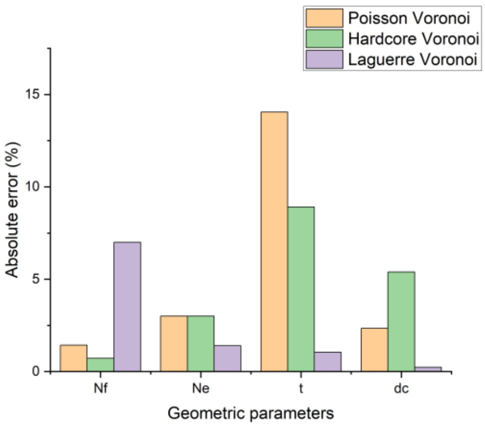

Fig. 9Absolute error in predicting the geometric parameters of the specimen

4.2. Mechanical Properties

The relative elastic modulus was calculated as the ratio of Young's modulus of the foam model to that of the bulk material.Table 2 presents the results.

Table 2: Relative modulus of the Voronoi-based models

Model	Relative Modulus E^*/E_s (%)
Specimen (Jang et al., 2015).	1.53
Poisson Voronoi	4.13
Hardcore Voronoi	1.92
Laguerre Voronoi	1.43

The results indicated that the Laguerre Voronoi models had the closest relative Young's modulus to the experimental results of the foam specimen.This is attributed to the ability of the Laguerre Voronoi tessellation to mimic the geometric properties of the real foam.These findings are consistent with those of earlier studies, such as that of Zhu et al. (2001), who investigated the effect of cell irregularity on the properties of 2D Voronoi honeycombs.Unlike the work of Zhu et al. (2001), this work considered Voronoi foams in 3D.

The absolute error in predicting the relative modulus of the specimen is shown in Fig. 10, which shows that the Laguerre Voronoi model has the smallest error of approximately 6.5%, whereas the Hardcore and Poisson Voronoi models have errors of 25% and 169%, respectively.Overall, these results suggest that the seed points generation method and position have a significant effect on the properties of the foam model. Therefore, researchers should pay attention when choosing the appropriate model for simulations. Based on the results the Laguerre Voronoi model is the most suitable for accurate prediction of the elastic properties.

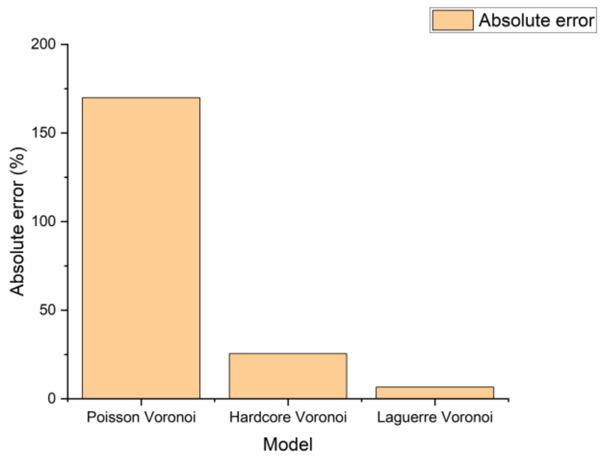

Fig. 10 Absolute error in predicting the relative modulus of the specimen

5. CONCLUSION

In this paper, an overview of various geometric modeling strategies for closed-cell foams is presented. The method of constructing the Kelvin model, Gibson and Ashby model, Poisson Voronoi, Hardcore Voronoi, and Laguerre Voronoi models were analyzed and discussed in terms of their geometric parameters. Additionally, the Voronoi-based models were meshed and subjected to a finite-element compressive test, and the results were compared with the experimental values of the specimens. The Laguerre Voronoi model was found to be the most accurate in predicting the elastic properties of the foam, with an absolute error of 6.5%. The Poisson Voronoi and Hardcore Voronoi models overestimated the elastic properties by 169% and 25%, respectively. Therefore, the Laguerre Voronoi model can be considered a suitable option for modeling closed-cell foams in future studies. These findings substantially add to our understanding of Voronoi foam modeling and highlight the importance of the seed point generation method in the construction of Voronoi foams.

REFERENCES

Andrews, E. W., & Gibson, L. J. (2001). The role of cellular structure in creep of two-dimensional cellular solids. *Materials Science and Engineering A*, *303*(1–2), 120–126. https://doi.org/10.1016/S0921-5093(00)01854-2

Bici, M., Campana, F., Università, S., Università, S., & Università, S. (2017). Mesoscale geometric modeling of cellular materials for Finite Element Analysis. *Computer Aided Design and Application*, *14*, 1–11.

Chen, Y., Das, R., & Battley, M. (2015). Effects of cell size and cell wall thickness variations on the stiffness of closed-cell foams. *International Journal of Solids and Structures*, *52*, 150–164. https://doi.org/10.1016/j.ijsolstr.2014.09.022

Falco, S., De Cola, F., & Petrinic, N. (2017). A method for the generation of 3D representative models of granular based materials. *International Journal for Numerical Methods in Engineering*, *112*(4), 338–359. https://doi.org/10.1002/nme.5521

Falco, S., Jiang, J., De Cola, F., & Petrinic, N. (2017). Generation of 3D polycrystalline microstructures with a conditioned Laguerre-Voronoi tessellation technique. *Computational Materials Science*, *136*, 20–28. https://doi.org/10.1016/j.commatsci.2017.04.018

Falco, S., Siegkas, P., Barbieri, E., & Petrinic, N. (2014). A new method for the generation of arbitrarily shaped 3D random polycrystalline domains. *Computational Mechanics*, *54*(6), 1447–1460. https://doi.org/10.1007/s00466-014-1068-3

Fritzen, F., Böhlke, T., & Schnack, E. (2009). Periodic three-dimensional mesh generation for crystalline aggregates based on Voronoi tessellations. *Computational Mechanics*, *43*(5), 701–713. https://doi.org/10.1007/s00466-008-0339-2

Gibson, L. J., & Ashby, M. F. (1982). The Mechanics of Three-Dimensional Cellular Materials. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, *382*(1782), 43–59. https://doi.org/10.1098/rspa.1982.0088

Gibson, L. J., & Ashby, M. F. (1997). *Cellular Solids: Structure and Properties*. Pergamon Press, Oxford.

Gong, L., Kyriakides, S., & Triantafyllidis, N. (2005). On the stability of Kelvin cell foams under compressive loads. *Journal of the Mechanics and Physics of Solids*, *53*(4), 771–794. https://doi.org/10.1016/J.JMPS.2004.10.007

Huang, J. S., & Gibson, L. J. (2003). Creep of open-cell Voronoi foams. *Materials Science and Engineering A*, *339*(1–2), 220–226. https://doi.org/10.1016/S0921-5093(02)00152-1

Jang, W.-Y., Hsieh, W.-Y., Miao, C.-C., & Yen, Y.-C. (2015). Microstructure and mechanical properties of ALPORAS closed-cell aluminium foam. *Materials Characterization*, *107*, 228–238. https://doi.org/10.1016/j.matchar.2015.07.012

Jeon, I., Asahina, T., Kang, K.-J., Im, S., & Lu, T. J. (2010). Finite element simulation of the plastic collapse of closed-cell aluminum foams with X-ray computed tomography. *Mechanics of Materials*, *42*(3), 227–236. https://doi.org/10.1016/J.MECHMAT.2010.01.003

Kloss, C., Goniva, C., Hager, A., Amberger, S., & Pirker, S. (2012). Models, algorithms and validation for opensource DEM and CFD-DEM. *Progress in Computational Fluid Dynamics, An International Journal*. https://doi.org/10.1504/PCFD.2012.047457

Li, Z., Zhang, J., Fan, J., Wang, Z., & Zhao, L. (2014). On crushing response of the three-dimensional closed-cell foam based on Voronoi model. *Mechanics of Materials*, *68*, 85–94. https://doi.org/10.1016/j.mechmat.2013.08.009

Nie, Z., Lin, Y., & Tong, Q. (2017a). Modeling structures of open cell foams. *Computational Materials Science*, *131*, 160–169. https://doi.org/10.1016/j.commatsci.2017.01.029

Nie, Z., Lin, Y., & Tong, Q. (2017b). Numerical investigation of pressure drop and heat transfer through open cell foams with 3D Laguerre-Voronoi model. *International Journal of Heat and Mass Transfer*, *113*, 819–839. https://doi.org/10.1016/j.ijheatmasstransfer.2017.05.119

Quey, R., Dawson, P. R., & Barbe, F. (2011). Large-scale 3D random polycrystals for the finite element method: Generation, meshing and remeshing. *Computer Methods in Applied Mechanics and Engineering*, *200*(17–20), 1729–1745. https://doi.org/10.1016/j.cma.2011.01.002

Redenbach, C., Shklyar, I., & Andrä, H. (2012). Laguerre tessellations for elastic stiffness simulations of closed foams with strongly varying cell sizes. *International Journal of Engineering Science*, *50*(1), 70–78. https://doi.org/10.1016/j.ijengsci.2011.09.002

Song, Y., Wang, Z., Zhao, L., & Luo, J. (2010). Dynamic crushing behavior of 3D closed-cell foams based on Voronoi random model. *Materials and Design*, *31*(9), 4281–4289. https://doi.org/10.1016/j.matdes.2010.04.007

Su, B. Y., Huang, C. M., Sheng, H., & Jang, W. Y. (2018). The effect of cell-size dispersity on the mechanical properties of closed-cell aluminum foam. *Materials Characterization*, *135*(April 2017), 203–213. https://doi.org/10.1016/j.matchar.2017.11.035

Tang, L., Shi, X., Zhang, L., Liu, Z., Jiang, Z., & Liu, Y. (2014). Effects of statistics of cell's size and shape irregularity on mechanical properties of 2D and 3D Voronoi foams. *Acta Mechanica*, *225*(4–5), 1361–1372. https://doi.org/10.1007/s00707-013-1054-4

Torquato, S., Gibiansky, L. V., Silva, M. J., & Gibson, L. J. (1998). Effective mechanical and transport properties of cellular solids. *International Journal of Mechanical Sciences*, *40*(1), 71–82. https://doi.org/10.1016/S0020-7403(97)00031-3

Weaire, D., & Phelan, R. (1994). A counter-example to Kelvin's conjecture on minimal surfaces. *Philosophical Magazine Letters*, *69*(2), 107–110. https://doi.org/10.1080/09500839408241577

Wejrzanowski, T., Skibinski, J., Szumbarski, J., & Kurzydlowski, K. J. (2013). Structure of foams modeled by Laguerre-Voronoi tessellations. *Computational Materials Science*, *67*, 216–221. https://doi.org/10.1016/j.commatsci.2012.08.046

Youssef, S., Maire, E., & Gaertner, R. (2005). Finite element modelling of the actual structure of cellular materials determined by X-ray tomography. *Acta Materialia*, *53*(3), 719–730. https://doi.org/10.1016/J.ACTAMAT.2004.10.024

Zhang, X., Tang, L., Liu, Z., Jiang, Z., Liu, Y., & Wu, Y. (2017). Yield properties of closed-cell aluminum foam under triaxial loadings by a 3D Voronoi model. *Mechanics of Materials*, *104*, 73–84. https://doi.org/10.1016/j.mechmat.2016.10.007

Zhang, X., Wu, Y., Tang, L., Liu, Z., Jiang, Z., Liu, Y., & Xi, H. (2016). Modeling and computing parameters of three-dimensional Voronoi models in nonlinear finite element simulation of closed-cell metallic foams. *Mechanics of Advanced Materials and Structures*, *6494*(June), 0–0. https://doi.org/10.1080/15376494.2016.1190426